Dilemma solving by the coevolution of networks and strategy in a 2×2 game

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A 2×2 game model implemented by a coevolution mechanism of both networks and strategy, inspired by the work of Zimmermann and Eguiluz [Phys. Rev. E **72**, 056118 (2005)] is established. Network adaptation is the manner in which an existing link between two agents is destroyed and how a new one is established to replace it. The strategy is defined as whether an agent offers cooperation (C) or defection (D). Both the networks and strategy are synchronously renovated in a simulation time step. A series of numerical experiments, considering various 2×2 game structures, reveals that the proposed coevolution mechanism can solve dilemmas in several game classes. The effect of solving a dilemma means mutual-cooperation reciprocity (R reciprocity), which is brought about by emerging several cooperative hub agents who have plenty of links. This effect can be primarily observed in game classes of the prisoner's dilemma and stag hunt. The coevolution mechanism, however, seems counterproductive for game classes of leader and hero, where the alternating reciprocity (ST reciprocity) is meaningful.

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I. INTRODUCTION

Sustainable cooperation in dilemma situations is one of the most interesting themes in various fields, such as biology, sociobiology [1], and other social sciences. An arising cooperation and its maintenance might be the most important key to understanding the social behavior of animals, such as mating competition and mutual altruism.

The reason for cooperation emergence in overcoming a dilemma has been primarily explained by two theories: kin selection [2] and reciprocal altruism [3]. Axelrod [4] insisted that the reason tit for tat (TFT) is effective in an iterated prisoner's dilemma (IPD) could be explained by direct reciprocity, where one could expect his opponent's cooperation (C) by offering his own C, instead of defection (D). Other options such as the not-participating option ("walk-away" or "lonely" strategy by, e.g., [5]) and discriminating option by means of tag (e.g., [6]) or reputation [7] rely on a mechanism, in which limiting opponents (depressing anonymity, in other words) leads to a rise in reciprocity. This can transform the game structure of "an egocentric action is the best way to maximize his own payoff" to a situation where the altruism is rather optimum (the altruism is more beneficial to himself in the long run, in other words).

The so-called spatial structures and social networking (classified as "network reciprocity" [8]) rely on the same principle. Hundreds of previous studies have examined "network reciprocity" (e.g., [9–15]). Network reciprocity can make altruism emerge, even though requiring that agents use only the simplest strategy—either C or D. Thus, network reciprocity may explain why a number of animal species, unsophisticated in terms of information processing, have evolved cooperative social systems. When we see the term "network," it encompasses various topologies from a regular or random graph, to a small-word [15] or a scale-free network [16]. Most of the earlier studies are based on a framework where agents are initially allocated in a fixed network. They play 2×2 games with opponents connected by network links. After gaming, they copy a strategy (defined by offering either C or D) from one of their neighbors, based on a certain rule. The copying process is either synchronous or asynchronous. In this series of procedures, adaptation is only considered in the process of strategy brush-up.

Hu *et al.* [17] and Tang *et al.* [18] report that a cooperative social structure, having a power-law payoff distribution, can emerge when both strategy adaptation and the growth of a scale-free network is taken account into the process of gaming either PD or chicken. This can be said to be one mechanism of coevolution of strategy and networking. Their model, however, assumes a particular process, where the network is growing (i.e., the number of agents is increasing), since the Barabasi-Albert algorithm [16] was involved in the process of dealing with a scale-free network.

Zimmermann and Eguiluz [19] showed the phenomenal idea of a perfect coevolution system in a networking game. The model they established can consider simultaneous coevolution of networks and strategy. Applying this model to several PDs, they observed a stable cooperation phase when a cooperative hub agent (they called a C leader) emerged, bringing about C chains. They argued that the detailed dynamics of an emerging cooperative phase could be described in terms of how the C chains are growing (or being broken). The fact of a C leader emerging as a hub agent implies that the emerging network in their model is similar to a scale-free network.

Another significant study, by Pacheco *et al.* [20,21], deals with both networking and strategy adaptations. They adopt two parameters: the time scale for strategy updating and one for network updating. When the former is smaller than the latter, this would be an evolutionary game in a fixed network. Assuming a complete graph as an initial network, this case can be analytically dealt with, using the replicator dynamics of a 2×2 game. They are analytically formulated for when the network updating scale is much less than that of strategy, which can be also evaluated by other replicator dynamics, using the a 2×2 game matrix, revised from the original.

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TABLE I. Payoff matrix for a 2×2 game.

		Opponent	
		Cooperation	Defect
Ego	Cooperation (C)	$\boldsymbol{R}$	$\boldsymbol{S}$
	Defect (D)	$\boldsymbol{T}$	$\boldsymbol{P}$

Their findings are not directly applicable to the case when both time scales seem to be close, which must be solved by a numerical approach, such as that used by Zimmermann and Eguiluz. In addition, the assumption of each agent having a complete graph as an initial state seems rather particular than usual, since the agent in a general social system has some countable links, determined by his finite information-processing capacity.

There is another recent study to be noted by Vainstein *et al.* [22]. They focus "mobility" in the context of a PD game on a spatially distributed population of memoryless, unconditional strategies (cooperators and defectors). The mobility, they say, can be interpreted as a network adaptation containing both severing and connecting a link. In other words, the network adaptation can be regarded as the agent's mobility in terms of biological application.

The present paper aims to establish a coevolution system in a networking game, which is primarily inspired by the model of Zimmermann and Eguiluz, but revised in some points concerning network adaptation. After a series of numerical experiments for various 2×2 game structures, we show that this coevolution of strategy and networking is effective to dilute dilemmas such as PD, stag hunt (SH), and chicken, where mutual cooperation (R reciprocity) is meaningful, but is ineffective for leader and hero dilemmas, where alternating reciprocity (where a focal agent offers C and opponent offers D, and they change roles in the next turn; ST reciprocity, in other words [23]) is more beneficial than R reciprocity.

II. MODEL

Let us presume a dynamic process, where an agent plays games with opponents connected by his networks (links) and evolves both his own strategy (either C or D) and the links to maximize his individual payoff. The number of agents in the society is N.

A. Game description

When we define respective payoffs in a 2×2 game as in Table I, every game defined in 2×2 game space can be expressed by [24]

$$P = x_0 - 0.5 r_1 \cos\left(\frac{\pi}{4}\right), \tag{1}$$

$$R = x_0 + 0.5 r_1 \cos\left(\frac{\pi}{4}\right), \tag{2}$$

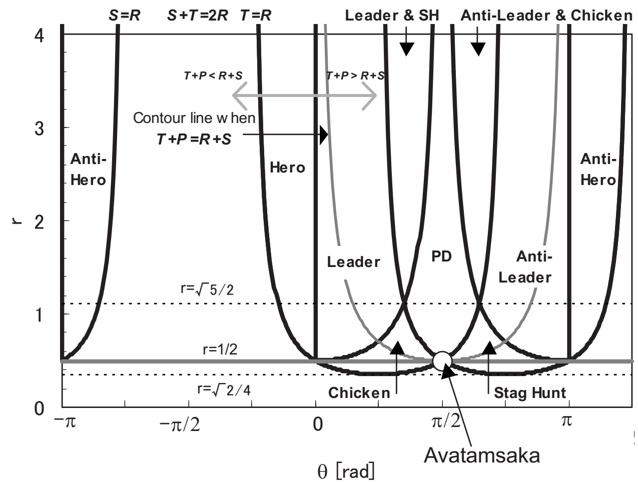

FIG. 1. Scene of the 2×2 game world. According to Tanimoto and Sagara [23], any 2×2 games can be parametrized by two game structural parameters r and θ. Any game classes including both dilemma games such as PD, chicken, SH and so on, and even trivial game, can be drawn schematically. Avatamsaka is a special game that is on the border of donor-recipient game (one of the special PDs) and trivial game.

$$S = x_0 + r_2 \cos\left(\frac{\pi}{4} + \theta\right), \tag{3}$$

$$T = x_0 + r_2 \sin\left(\frac{\pi}{4} + \theta\right). \tag{4}$$

Since x_0 is independent of the relative relationships among payoffs, a single set of two parameters $r \equiv r_2/r_1$ and θ [deg] is sufficient to see the entire 2×2 game world, as shown in Fig. 1. One marvelous feature is that well-known typical dilemma games, such as PD, chicken, SH, leader, and hero, can be distinctly drawn and the regions in which they occur can be illustrated. Tanimoto and Sagara [24] also show that the dilemma in a 2×2 game can be quantified with two game structural parameters $D_g = T - R$ and $D_r = P - S$. D_g indicates the static-dilemma intensity—the inclination of two equal players to exploit each other. They call this situation the gamble-intending dilemma (GID). Moreover, D_r indicates the static-dilemma intensity of equal players trying never to be exploited by each other. This is called a risk-averting dilemma (RAD). They also showed that the actual dilemma of a game can be explained by the game's structural (static elements D_g and D_r) and dynamic influences.

B. Agents

An agent makes both his strategy and network evolve to maximize his payoff. Those two adaptation processes operate synchronously. Each agent plays 2×2 games with all agents connected by his links. The total payoff defines the payoff, summing all games he plays at a certain time step. Hence, the more links an agent has, the higher payoff he can possibly earn. The average degree (average number of links) an agent has is denoted by K.

C. Strategy adaptation

Each agent deterministically copies the strategy (either C or D) from the neighbor (agents connected by his links) who obtained the biggest payoff in the previous time step. This is called imitation dynamics.

D. Network adaptation

At the beginning of the simulation episode, agents are connected by a random network (based on an Erdos-Renyi graph—e.g., [25]) having an average of K links. This random network remains for the initial 50 time steps. Thus, the agents play games in the fixed random network during the initial 50 time steps. After 50 steps, network adaptation starts. Network adaptation consists of two particular procedures: severing a link with one neighbor and connecting a new link to an unknown agent. We presume two methods for the two respective procedures. We define, here, probability p_k, which means that an agent keeps a certain link, never severing it spontaneously.

Severing method No. 1. Each agent tries to sever a D-D link by probability $1-p_k^2$. A link is never severed, unless both agents connected by the link offer D simultaneously. Namely, p_k means the probability in which an agent keeps its link in the next time step. An actual severing event must be a complementary event that two agents connected each other are trying to keep its link at the same time $(=1-p_k^2)$.

Severing method No. 2. Each agent tries to sever not only a D-D link but also a C-D link by probabilities $1-p_k^2$ (a probability of both two agents trying to serve) and $1-p_k$ (a probability of one of two agents trying to serve), respectively. An agent may sever a link whenever his opponent offers D. It seems plausible that the severing probability in case of both agents offering D is larger than when one agent offers D, because both agents try to sever the link in the former case, but in the latter case, only one tries.

Connecting method No. 1. Each agent who has severed a link creates a new link with an agent selected randomly from the population. The new link, however, is never the same as any existing link.

Connecting method No. 2. Each agent who has severed a link creates a new link with an agent proportionally selected through a roulette selection process, based on the average degrees of respective agents. On the grounds of the similarity to the Barabasi-Albert algorithm [16], this method may encourage a power-law degree distribution, like a scale-free network. The new link, however, can never be the same as an existing link.

III. NUMERICAL EXPERIMENT

The assumed experimental parameters are $K=8$, $N=1000$, $p_k=0.9985$ $(1-p_k^2=0.03)$, $r_1=1.272$, and $x_0=0.55$. Each initial distribution of C, imposed at the beginning state of every simulation episode, is assumed as 0.5. We vary the game structure $-\frac{3}{4}\pi \leq \theta \leq \frac{5}{4}\pi$ and $0 \leq r \leq 2$ in Eq. (1) (or Fig. 1). The contours shown below are drawn by ensemble averages of five equilibrium trials (quasi-steady-state of the dynamics) for respective game structures (we had confirmed that a five-ensemble average seems acceptable to observe the general tendency that will be discussed in the following text).

As far as the assumption of parameter setting, we have confirmed that the results below seem relatively robust, unless too large $1-p_k^2$ is assumed (too large $1-p_k^2$ leads to larger cooperation fraction for Leader game area).

One simulation episode runs until the time when the variations of cooperation fraction and payoff per an agents can be regarded sufficiently small after 2000 time steps, which seems an asymptotic equilibrium.

IV. RESULTS AND DISCUSSION

Figure 2 shows (a) the cooperation fraction among N in the case of an evolutionary network, based on severing method No. 1 and connecting method No. 1 [we call this the standard evolutionary network (SEN)]; (b) the payoff difference between SEN and the analytical solution (the raw payoff of the analytical solution can be accessed at Fig. 8 of [23]); (c) the payoff difference between SEN and the fixed random network case; (d) the payoff difference between SEN and the fixed scale-free network case; (e) the maximum degree of the network in SEN; (f) the payoff difference between the evolutionary network based on severing method No. 1 and connecting method No. 2 and the analytical solution; (g) the payoff difference between the evolutionary network based on severing method No. 2 and connecting method No. 1 and the analytical solution; and (h) the payoff difference between the evolutionary network based on severing method No. 2 and connecting method No. 2 and the analytical solution. Each payoff indicates a payoff per single game. The analytical solution comes from the replicator dynamics for a 2×2 game, which means the control case without any supporting cooperation mechanisms (such as game iteration, network, memory, punishment, etc.). Based on the analytical solution (e.g., [26]), every 2×2 game with infinite population size and none of supporting cooperation mechanisms can be classified whether it is C dominant (trivial game that contains no dilemma), D dominant (PD game having D_g and D_r at the same time), polymorphic (chicken-type dilemma game having D_g), or bistable (SH-type dilemma game having D_r). Both fixed random and scale-free networks indicate cases in which agents play 2×2 games in fixed networks that are determined at the beginning of a simulation episode. The payoff difference in (c) shows the effectiveness of network adaptation itself, because the initial state (up to 50 time steps) of the evolutionary network case is fixed to a random network.

Observing Figs. 2(a) and 2(b), we notice that the proposed coevolution system cannot solve stronger (larger r) chicken-type dilemma games [24], including the PD [enclosed by the dotted line in Fig. 2(a)]. However, it can solve most SH-type dilemma games and weak chicken-type dilemmas [24] [enclosed by the dashed line in Figs. 2(a) and 2(b)]. It also proves that, for leader and hero games (where ST reciprocity is beneficial than R reciprocity), coevolution obtains a smaller payoff [shown enclosed by the dot-dashed line in Fig. 2(b)] than the analytical solution. Hence, the coevolu-

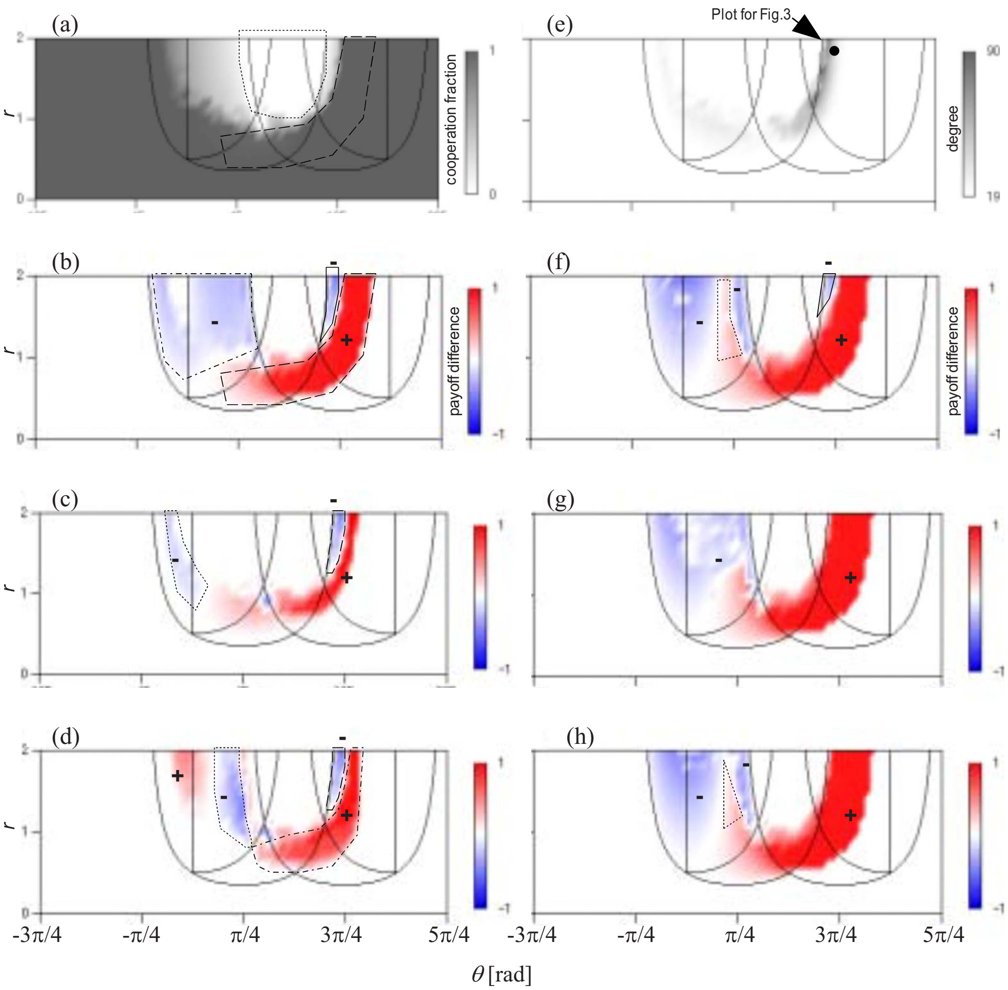

FIG. 2. (Color online) Result of the numerical experiments (a) the cooperation fraction among N in SEN, (b) the payoff difference between SEN and the analytical solution, (c) the payoff difference between SEN and the fixed random network, (d) the payoff difference between SEN and the fixed scale-free network, (e) the maximum degree in the SEN network, (f) the payoff difference between the evolutionary network based on severing method No. 1 and connecting method No. 2 and the analytical solution, (g) the payoff difference between the evolutionary network based on severing method No. 2 and connecting method No. 1 and the analytical solution, and (h) the payoff difference between the evolutionary network based on severing method No. 2 and connecting method No. 2, and the analytical solution. Each payoff indicates a payoff per single game. Degree distribution of the closed plot in (e) that locates on $\theta=3\pi/4$ and $r=1.8$ is shown in Fig. 3. Marks + and − indicate positive and negative differences, respectively.

tion system does not support ST reciprocity.

As we confirmed, the coevolution system can support R reciprocity by eliminating the dilemma to some extent. We can explain this by the emerging hub agents in the case of SEN that are produced by the coevolution system. Actually, the higher payoff area of SEN, compared with the fixed random network [the red area in Fig. 2(c)], is consistent with the area in which SEN has larger maximum degree [see Fig. 2(e)]. This seems consistent with what Zimmermann and Eguiluz [19] reported—that a particular degree distribution arises, like a scale-free network, due to the networks' adaptation. Hub agents can become C agents due to the evolutionary flexibility, directly produced by the strategy adaptation occurring simultaneously with the network adaptation. The cooperative hub agents that have more C agents in their subordinate layers emerge, which creates a C hierarchy structure, supporting R reciprocity, and leads to a higher payoff. Figure 3 shows one of the degree distributions of the closed plot (because the plot indicates an ensemble average) in Fig. 2(e) (bold line), of which the game structure is anti-leader ($\theta=3\pi/4$ and $r=1.8$ in Fig. 1) and that has a SH-type dilemma. In Fig. 3, both degree distributions of the fixed random network (thin line, the maximum degree is 19) and the fixed scale-free network (dotted line, the maximum degree is 74) are also shown. Obviously, SEN has a much larger maximum degree than the random network, almost that of the fixed scale-free network. Hence, SEN can enable the power-law-like degree distribution that the scale-free network has. Figure 4 shows degree distributions of both C agents and D agents in the case of SEN of Fig. 3, and Fig. 5 indicates payoff distributions of them. Figure 6 also deals with the case of SEN of Fig. 3, which shows payoff relations of two connected agents by a link; one is for the relation of the C-C link and the C-D link is another. We confirmed that

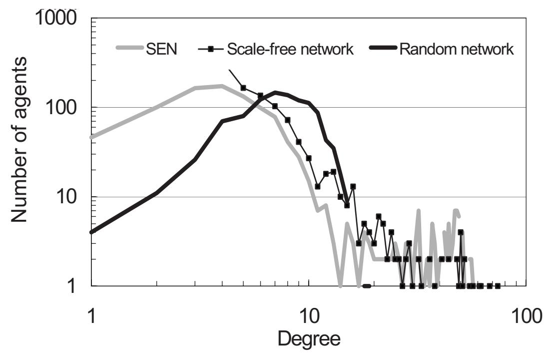

FIG. 3. Bold line indicates the degree distribution of the closed plot in Fig. 2(e) ($\theta=3\pi/4$ and $r=1.8$ in Fig. 1), where the game structure is antileader (see Fig. 1) with an SH-type dilemma. The thin and dotted lines are those of the fixed random network (maximum degree is 19) and the fixed, scale-free network (maximum degree is 74).

those results are robust by observing other degree distributions based on different random number seeds. Then, observing the results, we can notice that D agents who have fewer links in relation to C agents can only obtain modest payoffs featured by less deviation and less average than that of hub C agents. This proves that the "local max" (which is derived from the terminology of Zimmermann and Eguiluz [19]) D agents always exploit C agents who are in a middle position of a C chain (never be hub agents). As the total, the system can evolve to a sustainable cooperation, although several D agents are still alive. Because the hub agents who steer whether the society goes to cooperative or defective are C agents, and D agents manage to survive to exploit C agents who are not hubs but are in a position below the top.

Observing Fig. 2(c), we notice two areas where the payoff of SEN is less than the fixed random network. One is in either hero or leader (chicken-type dilemma), enclosed by a dotted line in Fig. 2(c); another is the area in antileader (SH-type dilemma), close to the PD border, enclosed by a dashed line in Fig. 2(c).

The first area can be explained by its particular game structure. Because it satisfies $2R<S+T$, in either hero or

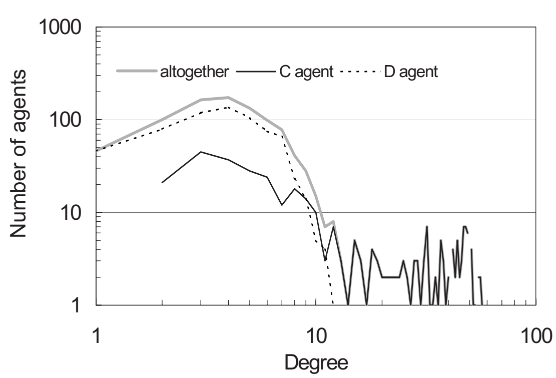

FIG. 4. Degree distributions of C agents, D agents, and entire agents in case of SEN. The game structure is same as Fig. 3 ($\theta=3\pi/4$ and $r=1.8$). The gray bold line ("altogether") is consistent with "SEN" in Fig. 3. The agent having the maximum degree 57 is a C agent. The maximum degree of D agents is 12.

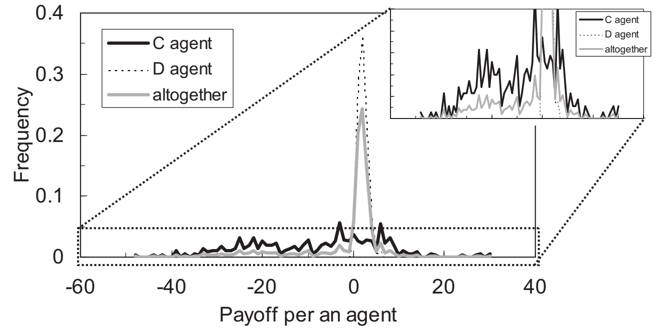

FIG. 5. Payoff (per an agent) distribution of C agents, D agents, and entire agents in case of SEN. The game structure is same as Fig. 3 ($\theta=3\pi/4$ and $r=1.8$).

leader, ST reciprocity is more beneficial than R reciprocity. In a fixed random network, unintentional ST reciprocity, by connecting a C agent with a D agent, happens to some extent. However, in SEN, this kind of situation is not allowed. When a focal agent offers C against a D opponent obtaining S, he might copy D strategy ($T>S$) from this opponent. If so, in the next time step, this link connects a D agent with a D agent. Network adaptation might sever the link. In this way, the network adaptation, plus strategy adaptation, leads this situation to R reciprocity, which leads to a smaller payoff.

In the second case, the game structure satisfies $2P>S+T$. Assuming an analytical solution, having no supporting cooperation mechanisms, antileader, in this area, leads to an all-defection state (every agent obtains P) if the initial C distribution is 0.5. Despite a game structure that encourages obtaining P, mutual defections become impossible, due to network adaptation, plus strategy adaptation. Namely, when a focal agent offers a C against a D opponent obtaining S, he retains C (because $S>T$), while a D-D link might be severed by network adaptation. In this way, there are always a certain number of C-D connections, instead of all "stable" D-D connections.

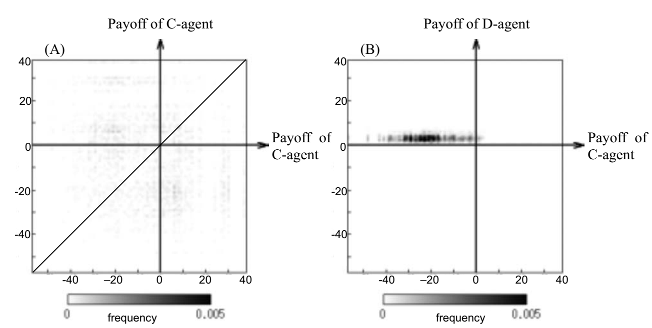

FIG. 6. Payoff distributions for two agents relations of both C-C link and C-D link in the case of SEN. The game structure is same as Fig. 3 ($\theta=3\pi/4$ and $r=1.8$). (A) is for C-C link distribution. X axis indicates a C agent's payoff. Y axis indicates another C agent's payoff who is connected with the first C agent. Hence this subgraph is asymmetric to the 45° line. (B) is for C-D link distribution. X axis indicates a C agent's payoff. Y axis indicates a D agent's payoff who has a link with the first C agent.

Figure 2(d) shows two areas where the payoff of SEN is less than that of the fixed scale-free network. One is the area in leader enclosed by a dotted line, and another is the area in antileader close to the PD border, enclosed by a dashed line. The latter area is consistent with one of the two inferior areas discussed previously [the dashed area in Fig. 2(c)]. The reason for this is the same as argued previously. The reason for the first low-payoff area can be that network adaptation cannot make adequate hubs evolve in this particular game structure. In fact, the maximum degree in the area [Fig. 2(e)] is less than the 74 available to the fixed scale-free network. A lack of hub agents who have larger degrees means that network adaptation cannot support R reciprocity effectively. Although low-payoff areas exist, we note that there is a high-payoff area starting at PD and stretching to parts of SH and antileader [enclosed by the dot-dashed line in Fig. 2(d)]. Hu *et al.* [17] reported that, for a fixed scale-free network, whether the maximum degree hub agent is assumed to be C or D at the beginning of a simulation episode crucially affects the following dynamics—almost determining whether the equilibrium would be a cooperative or defective phase. One reason the superior area stretching from PD to parts of SH and antileader emerges is that the proposed coevolution is effective in encouraging hub agents to become cooperative. Because of its flexibility, network adaptation can significantly help to extinguish defective hubs and enforce cooperative hubs emerging, which can lead the system to a stable R reciprocity. From the standpoint of Hu *et al.*, our result implies that coevolution, which considers simultaneous strategy and network adaptation, is a robust dilemma-solving mechanism, regardless of the influence of the initial-agent allocation that crucially affects fixed scale-free networks.

Comparing Figs. 2(b) and 2(f) and Figs. 2(g) and 2(h), connecting method No. 2 provides a higher payoff than the analytical solution in some parts of leader [enclosed by the dotted line in Figs. 2(f) and 2(h)]. As we have argued, the proposed coevolution seems less effective for ST reciprocity. However, adopting connecting method No. 2, instead of a random connection, works effectively in this leader area, even satisfying $2R < S+T$. Connecting method No. 2, which reflects an idea similar to the Barabasi-Albert algorithm, gives a network a larger maximum degree than does SEN, which can help more effective R reciprocity. Hence, it produces more R and less P fractions than either SEN or the analytical solution, which can realize a higher payoff.

Meanwhile, comparing Figs. 2(b) and 2(g) and Figs. 2(f) and 2(h), severing method No. 2 makes the low-payoff area in antileader, near the PD border [enclosed by the thick line in Figs. 2(b) and 2(f)] disappear. This is because it can sever not only a D-D link, but also a C-D link, which solves the particular drawback in which supporting R reciprocity inevitably brings S and T in a network, due to the condition of $2P > S+T$.

Summing up those two features, we would say that applying both connecting method No. 2 and severing method No. 2, which was not considered in the work of Zimmermann and Eguiluz, seems useful in providing more efficient reciprocity for several specific game structures.

V. CONCLUSIONS

We established a revised coevolution model of strategy and network adaptation from the original by Zimmermann and Eguiluz. After a series of investigation for various 2×2 game structures, we draw the following conclusions.

(i) This particular coevolution mechanism is useful for producing R reciprocity (valid for PD, chicken, and SH) and not significant for ST reciprocity (valid for leader and hero).

(ii) When we consider an additional network adaptation rule, "frequent connection to a larger degree agent," the coevolution mechanism produces a higher payoff than the analytical solution in some part of leader. This is because it fosters the emergence of hub agents having C strategy.

(iii) When we consider an additional network adaptation rule, "sever not only a D-D link but also a C-D link," the coevolution mechanism overcomes the drawback of less payoff than the standard network adaptation in some antileader, close to the PD border.

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